PRACTICE PROBLEMS FOR MIDTERM 1
MATH 430, SPRING 2014

Problem 1. *Determine if the following are tautologies:*
(a) $(R \to (S \vee Q)) \vee (R \vee (S \to Q))$
(b) $(R \leftrightarrow P) \vee (P \to \neg R)$

Problem 2. *Prove or refute the following:*
(a)If $\Sigma \models (\alpha \wedge \beta)$*, then* $\Sigma \models \alpha$ *and* $\Sigma \models \beta$
(b)If $\Sigma \models (\alpha \vee \beta)$*, then* $\Sigma \models \alpha$ *or* $\Sigma \models \beta$

Problem 3. *Let* S *be the set of all sentence symbols, and let* $\bar{S}$ *be the set of all (sentential) formulas built up from* S*. Fix a truth assignment* $\nu : S \to \{T, F\}$*. Without using the recursion theorem, show that there is at most one extension* $\bar{\nu} : \bar{S} \to \{T, F\}$ *satisfying the truth table conditions.*

Problem 4. *Show that* $\{\wedge, \leftrightarrow, +\}$ *is complete, but* $\{\wedge, +\}$ *is not complete. Here* $\alpha + \beta$ *means* $(\alpha \vee \beta) \wedge \neg(\alpha \wedge \beta)$ *i.e. either* α *or* β *is true, but not both.*

Problem 5. *Show that* $\{\perp, \to\}$ *is complete, but* $\{\wedge, \to\}$ *is not complete.*

Problem 6. *Show that* $\{\to, +\}$ *is complete, but* $\{\leftrightarrow, +\}$ *is not complete.*

Problem 7. *Recall that the corollary to the Compactness theorem states that if* $\Sigma \models \tau$*, then there is some finite* $\Delta \subset \Sigma$ *such that* $\Delta \models \tau$*. Show that the Compactness theorem is equivalent to this corollary. (Prove both directions.)*

Problem 8. *Suppose* Σ *is satisfiable and complete (i.e. for every formula* ϕ*, either* $\phi \in \Sigma$ *or* $\neg\phi \in \Sigma$*). Define a truth assignment* v *by* $v(A) = T$ *iff* $A \in \Sigma$*. Show that for each* $\phi \in \Sigma$*,* $\bar{v}(\phi) = T$ *iff* $\phi \in \Sigma$*.*

Problem 9. *Write the following in the first order language of set theory,* $\mathcal{L} = \{\in\}$*, as follows: first write down the sentence using symbols among* $\{\neg, \vee, \wedge, \to, \leftrightarrow, \forall, \exists, \in, =\}$ *and variables. Then rewrite the expression using only symbols among* $\{\neg, \to, \forall, \in, =\}$ *and variables (unless of course the already written expression is in this form).*
(a) x *is a subset of* y*.*
(b) The power set of x *is equal to the power set of* y*. (The power set of* x*, denoted* $\mathcal{P}(x)$*, is the set of all subsets of* x*.)*
(c) The well foundedness axiom: every nonempty set x *has an element* y*, such that no other element of* x *belongs to* y*.*

Problem 10. *Let* P *be a two-place predicate. Consider the formula:*

$$\phi = \text{"}\forall x \exists y P(x, y) \to \exists y \forall x P(x, y)\text{"}$$

(1) *Give an example of a model in which* ϕ *is false.*
(2) *Give an example of a model in which* ϕ *is true.*

(Note that when you describe the model, you have to say what is the interpretation of P*.)*